

Honors Algebra III/ Differential Calculus Summer Work

All work and answers should be on a separate sheet of paper. All work must be shown in order to receive full credit. If you worked with someone on these problems (friend, parent, sibling, tutor, etc), please indicate so on the top of your paper.

Factor completely and simplify.

1. $\frac{1}{4}x^2 - x - 48$

2. $8x^{-\frac{1}{2}} + 3x^{\frac{1}{2}}$

3. $4x^2 + 28x + 49$

4. $8x^3 - 1$

5. $x^3 + 3x^2 - 5x - 15$

6. $5(a - 1)^3 - 2(a - 1) + (a - 1)^2$

7. $6(x + 5)^{-2} + (x + 5)$

8. $(5w - 6)(w + 2) + (2w - 1)(w + 2)$

Simplify each expression. Make sure to rationalize any denominators.

9. $\frac{3+7i}{3-7i}$

10. $\sqrt[3]{54x^7y^{15}}$

11. $\frac{12}{\sqrt{7}-2}$

12. $\frac{15}{\sqrt{5}}$

13. $(-27)^{\frac{2}{3}}$

14. $\frac{x-3}{\sqrt{x+1}+2}$

15. $(6a^{\frac{2}{3}})(-4a^{\frac{3}{2}})$

16. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

17. $(9a^{\frac{5}{3}})^{\frac{3}{2}}$

18. Find the average of the two real numbers: $\frac{x}{2}$ and $\frac{x}{10}$

Find the inverse of the function and write as a function of x .

19. $y = x^2 + 10x + 25$

20. $y = \frac{x-4}{x+3}$

21. Given $f(x) = x^2 + 2x - 5$, find $f(x + h)$.

22. Find the point(s) of intersection between $x^2 - y^2 = 4$ and $y = 2x + 4$.

23. Simplify to a single expression: $\ln(x^2 - x) + \ln(x + 1) - \ln(x^2 - 1)$

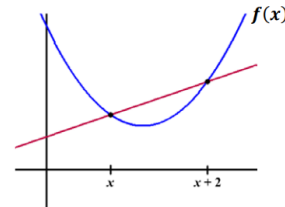
24. Use the graph to determine which expression represents the slope of the secant line. Choose one answer.

a) $\frac{f(x)-f(x+2)}{x+2-x}$

c) $\frac{f(x+2)-f(x)}{x+2-x}$

b) $\frac{f(x+2)-f(x)}{x-(x+2)}$

d) $\frac{x+2-x}{f(x)-f(x+2)}$



Find all the zeros (real and imaginary) of each function.

25. $f(x) = x^3 + x^2 - 4x - 4$

26. $f(x) = x^4 + 2x^3 + 3x^2 - 2x - 4$

27. $f(x) = 20x^3 - 500x$

28. $f(x) = x^4 - 13x^2 + 36$

29. Given the function $(x) = \frac{x^2-5x+6}{x^2-4}$, identify all asymptotes of the graph.

Solve each equation.

30. $2x^2 + x - 5 = 0$

31. $x^2 + 2 = -8x$

32. $x + 2 = \sqrt{2x + 7}$

33. $x^2 = -48$

34. $e^x + 1 = 2$

35. $\ln x = 0$

36. $e^{2x} - e^x = 0$

37. $\frac{100-4x}{3} = \frac{5x+6}{4} + 6$

38. The quadratic equation: $kx^2 + 4x + 2 = 0$ has two real roots. Find the maximum possible value of k .

Give the domain and range of each function in interval notation.

39. $f(x) = x^2 - 6x + 10$

40. $g(x) = \sqrt{2x - 5}$

41. $h(x) = \frac{3}{x-2} + 4$

42. $k(x) = \sqrt[3]{x+1} + 8$

Perform the indicated operation and simplify.

43. $\frac{2n^2-5n-12}{n^2-10n+24} \div \frac{4n^2-9}{n^2-9n+18}$

44. $\frac{3}{a-2} - \frac{1}{2-a}$

45. $\frac{1}{b^2-5b+6} + \frac{1}{b^2-4}$

46. $\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$

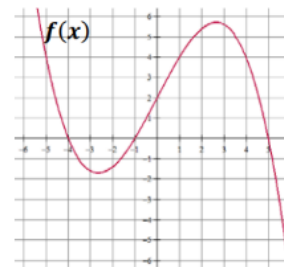
47. Given the volume of a cylinder: $V = \pi r^2 h$, write the formula for height in terms of volume if the radius of the cylinder is four times the height of the cylinder.

48. Sketch the graph of a continuous function with all of the following properties:

- a) $f(0) = 2$
- b) $f(-1) = f(4) = f(6) = 0$
- c) $f(x)$ is increasing on the interval $(-\infty, 2)$ and $(5, \infty)$
- d) $f(x)$ is decreasing on the interval $(2, 5)$

49. Use the graph of $f(x)$ to select any true statements from the list below. Choose all that apply.

- a) $f(2) = 4$
- b) $f(0) = 1$
- c) $(x - 5)$ is a factor of $f(x)$
- d) $f(1) = f(4)$



50. Use the graph of $g(x)$, defined from $[-3, 3]$, shown below to answer the following questions.

- a) Where is the graph positive?
- b) Where is the graph negative?
- c) Where is the graph increasing?
- d) Where is the graph decreasing?
- e) What are the zeros of the graph?
- f) What kind of symmetry does the graph have?
- g) Draw a sketch of $k(x)$ if $k(x) = -g(x - 3) + 2$.

